

Translation Variations

A striking difference between English and the formal language is the size of their vocabularies. While the formal language is lean to the point of dullness, English offers many different words to express the same idea. This is good news for poets, but a challenge when we need to shoehorn such linguistic extravagance into the cramped confines of Formalese. For English offers has not only a variety of names for a rose, but also for an “and” or “not”. These multiple English labels for the same logical concept are called **translation variations**. Before proceeding to thornier examples of logical form in English, we can quickly dispense with the minor complication posed by translation variations.

1. Negations. We know that English “not” is translated by the tilde, as in this example.

P: It is raining

It is **not** raining: $\sim P$

An obvious variation is the contracted form of “not”: “**n’t**”.¹ Another is the long-winded phrase “**it is not the case that**”.

P: It is raining

It is **not** raining: $\sim P$

It **isn’t** raining: $\sim P$

It is not the case that it is raining: $\sim P$

¹ This includes uses of “n’t” in the informal phrase “ain’t”. If “P” stands for “It is raining,” then “It **ain’t** raining” would be translated as “ $\sim P$ ”.

The words “**fail to**” also indicate negations – as the sameness of meaning in the following sentences illustrates.²

Q: It rained yesterday

It did **not** rain yesterday: ~Q

It **failed to** rain yesterday: ~Q

We can also make denials using fragments of English smaller than a word – the **negation morphemes**. Examples include “**im-**,” “**in-**,” “**un-**,” and “**il-**”.

The argument is valid: R

The argument is **invalid**: ~R

It is possible to travel faster than light: S

It is **impossible** to travel faster than light: ~S

I am able to jump 20 feet in the air: X

I am **unable** to jump 20 feet in the air: ~X

It is legal to drive 80 miles per hour on the interstate: T

It is **illegal** to drive 80 miles per hour on the interstate: ~T

Since “not” is only one way of denying a sentence in English, it isn’t accurate to call all these examples “**‘not’ sentences**”. While we could call them “denials” (and sometimes will, informally), our official name for such a sentence is a **negation**. In formal language, “~P” is the negation of “P”.

2. Conjunctions. Because “and” also has a number of translation variations in English, we likewise trade in the casual label “**‘and’ sentence**” for its technical counterpart “**conjunction**”. “It is sunny and it is cold” is an English conjunction, just as “(P ∧ Q)” is a conjunction of Formalese.

² A second, unrelated sense of “fail” is the opposite of “pass” – as in “Rex failed the quiz”. That sort of “fail” does *not* count as a variation on “not”.

An obvious variation on “and” is the two-word phrase “**both... and**”. Other variations are “**and also**,” “**and... too**,” “**and yet**,” and “**yet**”.

P: It’s cold

Q: It’s sunny

It’s cold **and** it’s sunny: $(P \wedge Q)$

It’s **both** cold **and** sunny: $(P \wedge Q)$

It’s cold, **and** it’s **also** sunny: $(P \wedge Q)$

It’s **cold**, and it’s sunny **too**: $(P \wedge Q)$

It’s cold, **and yet** it’s sunny: $(P \wedge Q)$

It’s cold, **yet** it’s sunny: $(P \wedge Q)$

The phrases “and yet” and “yet” carry an additional suggestion of incompatibility or opposition between the left and right parts of the conjunction. For example, saying “*It’s cold **yet** it’s sunny*” sounds alright, because sunny conditions tend to counteract cold ones. But “*Horses are muscular, **yet** they’re powerful*,” sounds strange, because being muscular isn’t opposed to being powerful. (By contrast, “*Horses are muscular **and** they’re powerful*” sounds fine, since “and” doesn’t convey a sense of opposition.)

All the same, “yet” and “and yet” assert both their left and right parts, just like “and”. So for logical purposes of truth and validity, the subtle differences in connotation aren’t a difference that makes a difference.

For the same reason we also count the phrases “**but**,” “**although**,” and “**even though**” as conjunction phrases, and translate them all as “ \wedge ”.

P: It’s sunny

Q: It’s cold

It’s cold **and** it’s sunny: $(Q \wedge P)$

It’s cold, **but** it’s sunny: $(Q \wedge P)$

It’s cold, **although** it’s sunny: $(Q \wedge P)$

It’s cold, **even though** it’s sunny: $(Q \wedge P)$

Finally, **relative clauses** can often be translated as conjunctions. A relative clause is a small sentence that is part of a larger sentence, and which describes something in the way an adjective does.

Ace is a **tall** engineer.

Ace is a **friendly** engineer.

Ace is an engineer **who likes rock climbing**

The adjectives “tall” and “friendly” describe what kind of engineer Ace is. But the relative clause “who likes rock climbing” describes in the same way: Ace is a ‘rock-climbing-liking’ sort of engineer. And a sentence with a relative clause makes the same double claim that a conjunction does.

Ace is an engineer **and** Ace likes rock climbing

Ace is an engineer **who likes rock climbing**

Each of these sentences asserts both of the following claims.

Ace is an engineer

Ace likes rock climbing

So we treat relative clauses as conjunctions in disguise. As the following examples show, we can replace the word at the beginning of the relative clause (“who,” “which,” “that”) with “and,” and patch up the remainder to match the left half of the conjunction. (For instance, the remainder of the relative clause “has a nice beach” is made a free-standing sentence by giving it the same subject – “Tel Aviv” – as the left half of the conjunction.)

Tel Aviv is a city [**that** has a nice beach]

Tel Aviv is a city, **and** Tel Aviv has a nice beach

The Umbrellas of Cherbourg is a movie [**which** everyone should see]

The Umbrellas of Cherbourg is a movie, **and** everyone should see

The Umbrellas of Cherbourg

3. Disjunctions. We replace the informal label “**or** sentence” with the more technical term “**disjunction**”.

Just as “and” could appear with or without its left partner “both,” so “**either... or**” says the same as simple “**or**”. Less obviously, “**unless**” also serves as a translation variation.

P: Rex is at home

Q: Rex is at the store

Either Rex is home **or** he’s at the store $(P \vee Q)$

Rex is home **or** he’s at the store $(P \vee Q)$

Rex is at home **unless** he’s at the store $(P \vee Q)$

English usage allows these phrases to express two quite different types of disjunction. For instance, if the daily special in a restaurant includes soup **or** salad, and a customer choosing that special expects both soup **and** salad included, he has misinterpreted the “or” intended. For that one low price, the special offers soup **or** salad **but not both**; getting both costs extra. This sort of disjunction – offering one option or the other, but not both – is an **exclusive “or”**.

By contrast, if the cost of the special includes coffee, and the customer takes his with both cream and sugar, he needn’t worry about added expense when asked if he wants “cream **or** sugar”. Here the price of the special includes cream, or sugar, **or both**. This is an **inclusive “or”**.

“Unless” is often (perhaps typically) exclusive. So when we say

We will have a picnic **unless** it rains

the sentence is clearly intended to rule out having both (that is: a picnic in the rain).

But my warning

You will fail logic class **unless** you study

doesn't rule out the possibility of your both studying *and* failing the class – if, for example, you study hard but don't show up for the exams. Since we don't find such an inclusive/exclusive ambiguity in negation or conjunction phrases, the fact that “unless” permits both readings is evidence that it's a translation variation of “or”.

We follow logical tradition in giving the vel an **inclusive** reading. So

$(P \vee Q)$

means

P, or Q, or possibly both.

English wording provides an abbreviated guide to stating an *exclusive* disjunction in the formal language. Since “both” always accompanies an “and,” the exclusive “or” sentence

P or Q but not **both**

is short for

P or Q but not **both P and Q**.

Recognizing “but” as a variation on “and,” we see how to translate the whole sentence.

P or Q **but** not both P and Q
 $((P \vee Q) \wedge \sim (P \wedge Q))$

Often we must appeal to conversational context, and the meaning of the sentences being disjoined, to determine whether an inclusive or exclusive disjunction is intended. But our policy in this book keeps matters simple: we will translate English disjunctions as **inclusive, unless clearly indicated otherwise** (by the speaker saying “but not both,” or something equally unambiguous).

4. More Complex Form Phrases. So far we have translated each English form phrase with a single connective. But for some form phrases that doesn't work.

“**Neither... nor**” is one example. It walks and talks like a form phrase, providing argument patterns that remain valid regardless of the subject matter. Here is one example.

$$\frac{\text{Neither } \bullet \text{ nor } \blacktriangle .}{\therefore \text{Not } \bullet .}$$

And here are some intuitively valid English instances of this pattern.

$$\frac{\text{We're having } \mathbf{neither} \text{ ice cream } \mathbf{nor} \text{ cake.}}{\therefore \text{We're } \mathbf{not} \text{ having ice cream.}}$$
$$\frac{\text{Logic is } \mathbf{neither} \text{ difficult } \mathbf{nor} \text{ boring.}}{\therefore \text{Logic is } \mathbf{not} \text{ difficult.}}$$

But no single tilde, wedge, or vel will properly translate this phrase into our formal language.

We then face two options. We could introduce another connective into the formal language, as the formal counterpart to “neither... nor”. That would keep translation simple, at the cost of complicating the formal language.

A clue toward the second option comes in recognizing two familiar bits of form in “neither... nor”: “either... or” and the “n” of negation. And in fact a “neither... nor” has the same meaning as the *denial* of an “either... or”.

Q: Are we having either ice cream or cake?
A1: No, we're **not** having **either**.
A2: No, we're having **neither** ice cream nor cake.

A formal negation of a disjunction will thus translate “neither... nor”.

P: We’re having ice cream

Q: We’re having cake.

We’re having **neither** ice cream **nor** cake.

$$\sim(P \vee Q)$$

Our translation methods follow this second course: restricting the formal language to just three connectives, and translating “neither... nor” as a combination of two connectives.

“Neither... Nor” and Conjunctions

It occurs rather naturally to English speakers that the following two sentences also seem equivalent in meaning.

“We’re having **neither** ice cream **nor** cake.”

“We’re **not** having ice cream **and** we’re **not** having cake.”

Using the same translation table as before, this second sentence is translated as follows.

$$(\sim P \wedge \sim Q)$$

And this suggests that our formal translation of “neither... nor” is equivalent to this conjunction, just as the above two English sentences are equivalent.

$$\sim(P \vee Q)$$

$$(\sim P \wedge \sim Q)$$

As a matter of fact that’s true – as we’ll later show, using various formal methods.

Still, I resist translating a “neither... nor” sentence by the conjunction of negations, for a simple reason: the negation of a disjunction still mirrors the English grammar a little more faithfully. In “ $\sim(P \vee Q)$ ” we see both disjunction and negation, just as we earlier did in “neither... nor”.

So our **official translation of “neither... nor”** will be as the negation of a disjunction: “ $\sim(P \vee Q)$ ”

The phrase “**without**” likewise provides argument patterns valid regardless of subject matter.

● without ▲

∴ ●

● without ▲

∴ Not ▲

For instance, both of these English arguments are valid.

1. Rex passed Chemistry without doing the labs.

∴ Rex passed Chemistry.

1. Rex passed Chemistry without doing the labs.

∴ Rex did not do the labs.

We could add a new connective to translate “without”. But the same two-connective maneuver serves here. Note that the valid patterns we see in “without” also hold for a conjunction with a negated right part.

$\frac{(\bullet \wedge \sim \blacktriangle)}{\therefore \bullet}$	$\frac{\bullet \text{ without } \blacktriangle}{\therefore \bullet}$
$\frac{(\bullet \wedge \sim \blacktriangle)}{\therefore \sim \blacktriangle}$	$\frac{\bullet \text{ without } \blacktriangle}{\therefore \sim \blacktriangle}$

That suggests “*P without Q*” be translated as “ $(P \wedge \sim Q)$ ”. Further support comes from the intuitive *sameness of meaning* between a “without” sentence and a conjunction with negated right half.

Rex passed Chemistry **without** doing the labs
 Rex passed Chemistry **even though** he didn’t do the labs

Ace passed me **without** saying “Hi”
 Ace passed me **but** he didn’t say “Hi” .

So we translate “without” as the two-connective cluster “ $\wedge \sim$ ”.

P: Rex passed Chemistry
 Q: Rex did the labs

“Rex passed Chemistry **without** doing the labs.”
 $(P \wedge \sim Q)$

Reconstructing the Right Side of a “Without” Sentence

Notice that we must reconstruct the right part of the “without” sentence to get a normal subject matter sentence in our translation table.

R: Ace passed me

S: Ace said “Hi”

Ace passed me without *saying* “**Hi**”

“**Saying ‘Hi’**” is a compressed mini-sentence. To recover the full sentence “*Ace said ‘Hi’*” for the translation table, we execute these three steps.

First, since this collapsed sentence lacks a subject, give it the same subject appearing in the left subject matter sentence.

Ace passed me

Ace saying “Hi”

Second, clip the “-ing” from the verb.

Ace say “Hi”

Third: have the verb discuss the same **time** that the left sentence does. Here the left sentence discusses the past (as the “-ed” in “passed” makes clear); so the right sentence does the same.

Ace passed~~ed~~ me

Ace said “Hi”.

This ‘re-inflated’ sentence is suitable for a translation table.

R: Ace passed me

S: **Ace said “Hi”**

A further cautionary note is in order here: while this translation strategy works reasonably well, the **denial** of a “without” sentence brings a further complication. Consider an ordinary English conjunction.

Rex passed Chemistry and didn’t do the labs.

Denying just the left part doesn’t, of course, deny the whole sentence.

Rex **didn’t** pass Chemistry and didn’t do the labs

To deny the entire conjunction we add “it is not the case that” to its left.

It is not the case that [Rex passed Chemistry but didn’t do the labs].

But though a “without” sentence behaves like a conjunction, here we deny the *entire sentence* in English when negating its **left part**.

“Without” Sentence: Rex passed Chemistry without doing the labs.

Denial: Rex **didn’t** pass Chemistry without doing the labs

P: Rex passed Chemistry

Q: Rex did the labs

Rex passed Chemistry without doing the labs.

$(P \wedge \sim Q)$

Rex **didn’t** pass Chemistry without doing the labs.

$\sim(P \wedge \sim Q)$.

Don’t blame the formal language for this. It’s strictly a peculiarity of English.

But it's not a peculiarity restricted to "without" sentences, as the same pattern appears in relative clauses – another English "conjunction in disguise".³

T: Ace is an engineer

U: Ace likes rock climbing

Ace is an engineer who likes rock climbing

$(T \wedge U)$

Here too, even when adding a negation phrase to just the left half of the English sentence we wind up denying the sentence in its entirety.

Ace is **not** an engineer who likes rock climbing

$\sim(T \wedge U)$

True, we can always appeal to the less-confusing "it is not the case that" – which is visibly attached to the entire sentence. But we must keep in mind this bit of English oddness.

Think of "without" and relative clause sentences as wrapped in parentheses which a negation phrase cannot penetrate – thereby permitting negation *only* of the sentence as a whole.

It is not the case that [Rex passed Chemistry without doing the labs]
= Rex **didn't** pass Chemistry without doing the labs

$\sim(P \wedge \sim Q)$

It is not the case that [Ace is an engineer who likes rock climbing]
= Ace is **not** an engineer who likes rock climbing

$\sim(T \wedge U)$

³ And note that translating a relative clause sentence requires reconstructing its right half, just as a "without" sentences does.

Summary: More Complex Form Phrases

- *Neither P nor Q:*
 $\sim(P \vee Q)$
- *P without Q:*
 $(P \wedge \sim Q)$

Reconstructing the Right Half of a 'Without' Sentence:

- Add the same subject used in the left half of the “without” sentence.
- Delete “-ing” from the verb
- Make the verb discuss the same time as the left sentence.